where

$$F(n) = \left[\frac{0.0138(n-1)^3(n^2+3)}{n^2(n+1)(n-2)^2}\right]^{1/3} \times (T-1)\left[\frac{1}{T^2}\left\{1+\frac{3(n^2-1)T}{(n^2+3)}\right\}\right]^{1/3} \quad (22)$$

When there is uniform suction downstream of the leading edge of flat plate, then Eq. (22) becomes

$$\xi^{1/2} = F(n) \quad (23)$$

where

$$\xi = w_0{}^2 x/u_0 v \quad (24)$$

The result calculated from Eq. (23) is given in Table 2 together with Iglisch exact solution and Curle approximate solution for the purpose of comparison. The comparison in Table 2 shows that the present approximate method produces results which are slightly more accurate than those given by Curle.

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On Statistical Analysis of Composite Solid Propellant Combustion

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HERMANCE[1] and Beckstead, Derr, and Price[2] have formulated steady-state combustion models that employ statistical concepts. This is an important advance because it leads to an "explicit" and calculatable relation between oxidizer particle size and burning rate. The objectives of this Note are first to show that the statistical portions of these analyses, which are virtually identical, are questionable and second to reformulate the problem "correctly" for the assumed burning surface topography. The reformulation is a generalization of the pioneering work of Miller, Hartman, and Myers.[3]

Received June 13, 1973; revision received October 2, 1973.

Index categories: Combustion in Heterogeneous Media; Solid and Hybrid Rocket Engines.

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† Flamelet is employed to denote the complex multi-flame structure associated with specific oxidizer/fuel surface pairs.

‡ Nomenclature is the same as that of Ref. 2.

The burning surface of a composite solid propellant is an ensemble (like a patch work quilt) of different flamelets.† In Refs. 1 and 2 the ensemble is replaced with a single flamelet and the burning surface topography is assumed to be a fuel plane dotted with concave and/or convex oxidizer surfaces. The characteristic spatial dimension(s) of the single flamelet is based on the mean oxidizer particle intersection area of a planar "cut" through a unimodal propellant with randomly packed spherical oxidizer particles. In such a "cut" the oxidizer particle intersection areas range from zero to $\pi D_o{}^2/4$‡ and the mean intersection area is $\pi D_o{}^2/6$. It is important to note that the characteristic dimension is based solely on geometry.

The Hermance and Beckstead, Derr, and Price approaches are questionable on the following two points: 1) the physical validity of replacing the behavior of an ensemble of different flames with that of a single flame (a basic assumption in most quasi-one-dimensional analyses of composite solid propellant combustion), and 2) the statistical process employed to select the characteristic dimension. Both points are related. The total oxidizer vapor flow from the burning surface M_{ox} is the sum of the oxidizer flows from all of the individual oxidizer surfaces so

$$M_{ox} = \sum_i m_{ox,i}\,\varepsilon_{ox,i} \quad (1)$$

where $m_{ox,i}$ is the flux of oxidizer from and $\varepsilon_{ox,i}$ the area of the ith oxidizer surface. For quasi-steady conditions the mixture ratio is fixed by the composition of the solid. Therefore

$$\bar{m}_T = \sum_i m_{ox,i}\,\varepsilon_{ox,i}/(\alpha S_o) \quad (2)$$

Numerical results from both Refs. 1 and 2 show that, in general, $m_{ox,i}$ is a function of $\varepsilon_{ox,i}$. Consequently, a characteristic dimension selected from geometry alone is not compatible with Eq. (2) so that the statistical aspects of Refs. 1 and 2 appear to violate continuity. It should be noted that criticism is not directed at the physiochemical combustion models.

The reason for difficulty is clear. The statistical aspects must be compatible with Eq. (2). Let ε_f and ε_{ox} be the fuel and oxidizer surfaces associated with a flamelet and F_{ox} and F_f be distribution functions such that the fraction of oxidizer surfaces with $\varepsilon_{ox} \leqq \varepsilon_{ox} \leqq \varepsilon_{ox}+d\varepsilon_{ox}$ is $F_{ox}\,d\varepsilon_{ox}$ and the fraction of fuel surfaces with $\varepsilon_f \leqq \varepsilon_f \leqq \varepsilon_f+d\varepsilon_f$ is $F_f\,d\varepsilon_f$. Then, if N is the number of oxidizer surfaces per unit planar area of burning surface, the number of flames with $\varepsilon_{ox} \leqq \varepsilon_{ox} \leqq \varepsilon_{ox}+d\varepsilon_{ox}$ is

$$dN = NF_{ox}\,d\varepsilon_{ox} \quad (3)$$

Since the solid propellant is a random packing, assume that the distribution functions are independent.§ Consequently, the fraction of the dN flamelets with $\varepsilon_f \leqq \varepsilon_f \leqq \varepsilon_f+d\varepsilon_f$ is

$$d^2N = NF_f F_{ox}\,d\varepsilon_f \quad (4)$$

If it is further assumed that the individual flamelets are non-interacting (each flamelet does its own thing), the mass flux of oxidizer from each oxidizer surface becomes a function of pressure, intial propellant temperature, freestream velocity, oxidizer and fuel surface area, etc. The mass flux of oxidizer from flamelets with $\varepsilon_{ox} \leqq \varepsilon_{ox} \leqq \varepsilon_{ox}+d\varepsilon_{ox}$ and $\varepsilon_f \leqq \varepsilon_f \leqq \varepsilon_f+d\varepsilon_f$ is

$$d^2\bar{m}_{ox} = Nm_{ox}\varepsilon_{ox}F_{ox}F_f\,d\varepsilon_{ox}\,d\varepsilon_f \quad (5)$$

The mass flow of oxidizer from flamelets with $\varepsilon_{ox} \leqq \varepsilon_{ox} \leqq \varepsilon_{ox}+d\varepsilon_{ox}$ is obtained by "summing" over all possible ε_f so

$$d\bar{m}_{ox} = NF_{ox}\varepsilon_{ox}\left(\int_{\varepsilon'_f}^{\infty} m_{ox}F_f\,d\varepsilon_f\right)d\varepsilon_{ox} \quad (6)$$

The infinite upper limit occurs because there is no specific physical upper bound to ε_f. Of course, the probability of large ε_f is vanishingly small.¶ The finite lower limit occurs because oxidizer particles cannot interpenetrate. The mean mass flux of

§ Although this assumption seems reasonable, further study is warranted. It is a complex question because detailed knowledge of the statistical nature of the random packing is required.

¶ Note the similarity here to the kinetic theory of gases where each velocity component ranges from $-\infty$ to ∞.

oxidizer from the burning surface is then the integral of Eq. (6) over all possible ε_{ox} or

$$\bar{m}_{ox} = N \int_0^{\varepsilon_{ox}''} F_{ox}\, \varepsilon_{ox} \int_{\varepsilon_f'}^{\infty} m_{ox} F_f \, d\varepsilon_f \, d\varepsilon_{ox} \qquad (7)$$

where ε_{ox}'' is the largest possible oxidizer surface. Since mixture ratio is preserved with means

$$\bar{m}_T = \bar{m}_{ox}/\alpha \qquad (8)$$

The statistical formulation presented indicates that statistical combustion modeling is more complex than the analyses of Hermance and Beckstead, Derr, and Price suggest. However, the increase in complexity may be worthwhile because a basic assumption in one-dimensional combustion analyses of composite solid propellant combustion—substitution of a single flamelet for an ensemble—has been removed. In addition, virtually any one-dimensional combustion model may be employed to relate m_{ox} to its independent variables. Therefore, this formulation rather than degrading existing one-dimensional models provides a framework that may enhance their physical realism. Furthermore, there is a subtle but very powerful advantage to this statistical approach. With each flamelet "doing its own thing" in response to external stimuli and the response(s) to that stimuli determinable at the individual flamelet level through quasi-one-dimensional combustion models, flamelets favorable to and detrimental to a desired response can be identified. Consequently, desired responses can be enhanced by reducing the population of detrimental flamelets and augmenting the population of favorable flamelets. Therefore, this statistical formulation provides a systematic (and potentially quantitative) basis for the control of combustion phenomena through the manipulation of oxidizer particle sizes.

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Stability of Two-Hinged Circular Arches with Independent Loading Parameters

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Nomenclature

a = constant radius of the undeformed centroidal line of the arch rib
EI = bending stiffness
P = downward point load acting at the crown of an arch on the verge of buckling
$P_o = P$ when $w = 0$
W = total weight of the arch rib
$w = W/2a\alpha$
w_o = critical own weight when $P = 0$
v_c = vertical displacement of the crown of the arch
2α = subtending angle of the arch

Received June 21, 1973; revision received October 10, 1973. This investigation was supported by National Science Foundation Grant GK-19726.

Index category: Structural Stability Analysis.

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WHEN nonshallow arches buckle at large deflections, their own deadweight is usually not a negligible quantity as compared to the applied load, and the problem of calculating critical loads becomes a two-parameter nonlinear boundary-value problem. Since it is very difficult to eliminate the effect of own weight in experimental work, it is of considerable importance to determine relationships between the two loads at the instant of impending instability of the arch.

For the foregoing reason, it was decided to calculate interaction curves for the critical values of the downward point load P at the crown of two-hinged circular arches and the uniformly distributed own weight w per unit length of the centroidal line. The exact nonlinear theory of the inextensible elastica was used in the calculations. This theory yields excellent results in the case of nonshallow slender arches.[1,2]

The governing equations of the theory of the initially curved elastica will not be derived herein, as they can be found in published literature.[3-5] A brief summary of the necessary equations is presented in Ref. 6. These equations were solved by means of an electronic digital computer to a high degree of accuracy. It was found that arches with subtending angles in the range $2\alpha = 60°$–$270°$ (see Fig. 1 of Ref. 6) buckle asymmetrically by sidesway, a bifurcation type of buckling.

The calculated critical values P and W of the interacting point load and the total weight of the arch, respectively, are made dimensionless by dividing them by the critical values P_o and W_o of these forces acting singly, i.e., $P = P_o$ when $W = 0$ and $W = W_o$ when $P = 0$. The P/P_o vs W/W_o interaction curves appear nearly straight (e.g., see Fig. 1) and can be approximated accurately by the simple equation

$$(P/P_o) + (W/W_o) = 1 \qquad (1)$$

for all the values of the subtending angle 2α for which calculations have been carried out, namely, for $\alpha = 30°$, $40°$, $50°$, $60°$, $70°$, $80°$, $90°$, $100°$, $110°$, $135°$. No consistency in regard to the convexity or concavity of the interaction curves is apparent; the curves for $\alpha = 30°$, $40°$, $50°$, $135°$ are very slightly convex away

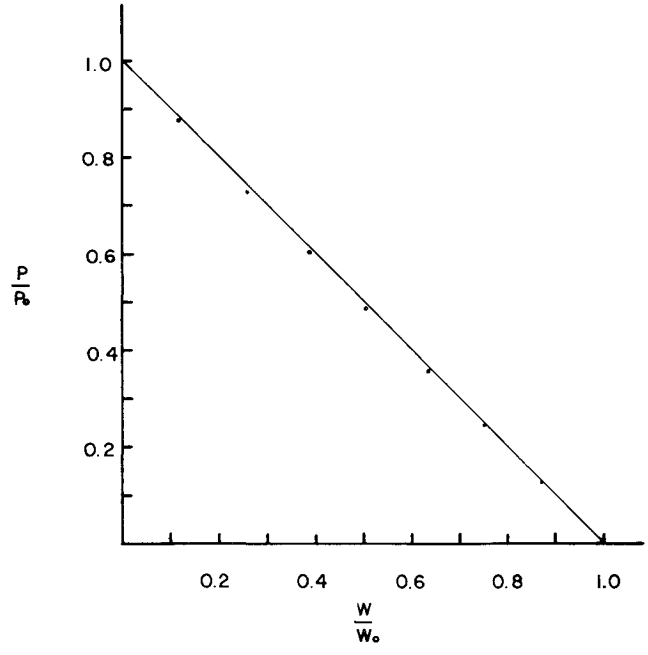

Fig. 1 Interaction curve for $\alpha = 80°$ (dots represent calculated points).